Your Name ____

Wants to retake the Chapter 3 test on Parallel Lines and Transversals

Original Score ___________

Before you take the test:

_____ Do test corrections on your original Ch 3 Test

_____ Print and complete the review packet

_____ Attend lunch help with Ms. Biller or at the math center

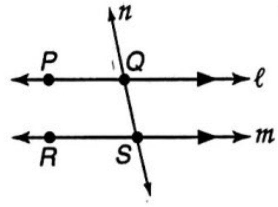
_____ Parent Signature ___________________________

Bring completed packet to retake the test on Thursday Dec. 7th.

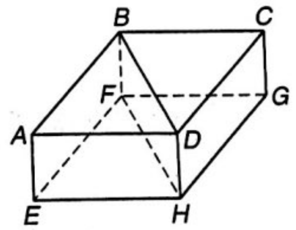
3-1 Study Guide and Intervention

Parallel Lines and Transversals

Relationships Between Lines and Planes When two lines lie in the same plane and do not intersect, they are **parallel**. Lines that do not intersect and are not coplanar are **skew lines**. In the figure, ℓ is parallel to m, or $\ell \parallel m$. You can also write $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$. Similarly, if two planes do not intersect, they are **parallel planes**.



Example



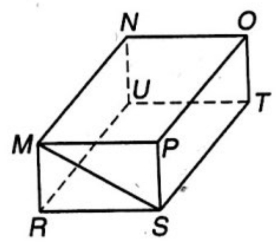
a. Name all planes that are parallel to plane *ABD*.
plane *EFH*

b. Name all segments that are parallel to $\overline{CG}$.
$\overline{BF}$, $\overline{DH}$, and $\overline{AE}$

c. Name all segments that are skew to $\overline{EH}$.
$\overline{BF}$, $\overline{CG}$, $\overline{BD}$, $\overline{CD}$, and $\overline{AB}$

Exercises

For Exercises 1–3, refer to the figure at the right.

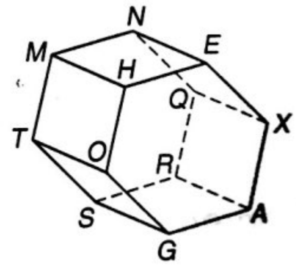


1. Name all planes that intersect plane *OPT*.

2. Name all segments that are parallel to $\overline{NU}$.

3. Name all segments that intersect $\overline{MP}$.

For Exercises 4–7, refer to the figure at the right.



4. Name all segments parallel to $\overline{QX}$.

5. Name all planes that intersect plane *MHE*.

6. Name all segments parallel to $\overline{QR}$.

7. Name all segments skew to $\overline{AG}$.

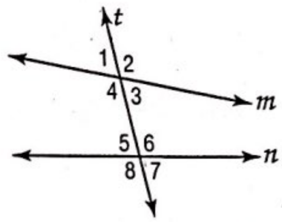
NAME ______________________ DATE ________ PERIOD ____

3-1 Study Guide and Intervention *(continued)*

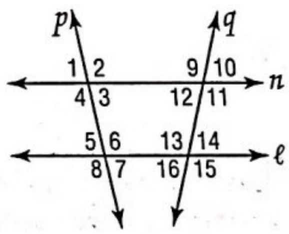
Parallel Lines and Transversals

Angle Relationships A line that intersects two or more other lines in a plane is called a **transversal**. In the figure below, t is a transversal. Two lines and a transversal form eight angles. Some pairs of the angles have special names. The following chart lists the pairs of angles and their names.

Angle Pairs	Name
∠3, ∠4, ∠5, and ∠6	interior angles
∠3 and ∠5; ∠4 and ∠6	alternate interior angles
∠3 and ∠6; ∠4 and ∠5	consecutive interior angles
∠1, ∠2, ∠7, and ∠8	exterior angles
∠1 and ∠7; ∠2 and ∠8	alternate exterior angles
∠1 and ∠5; ∠2 and ∠6; ∠3 and ∠7; ∠4 and ∠8	corresponding angles



Example **Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.**



a. **∠10 and ∠16**
alternate exterior angles

b. **∠4 and ∠12**
corresponding angles

c. **∠12 and ∠13**
consecutive interior angles

d. **∠3 and ∠9**
alternate interior angles

Exercises

Use the figure in the Example for Exercises 1–12.

Name the transversal that forms each pair of angles.

1. ∠9 and ∠13

2. ∠5 and ∠14

3. ∠4 and ∠6

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

4. ∠1 and ∠5

5. ∠6 and ∠14

6. ∠2 and ∠8

7. ∠3 and ∠11

8. ∠12 and ∠3

9. ∠4 and ∠6

10. ∠6 and ∠16

11. ∠11 and ∠14

12. ∠10 and ∠16

NAME ______________________ DATE __________ PERIOD ____

3-2 Study Guide and Intervention

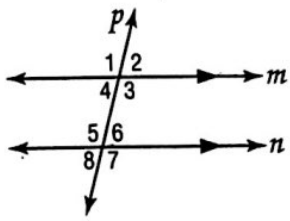
Angles and Parallel Lines

Parallel Lines and Angle Pairs When two parallel lines are cut by a transversal, the following pairs of angles are congruent.

- corresponding angles
- alternate interior angles
- alternate exterior angles

Also, consecutive interior angles are supplementary.

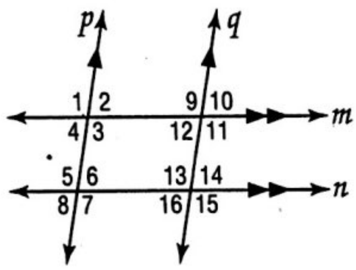
Example **In the figure, $m\angle 2 = 75$. Find the measures of the remaining angles.**



$m\angle 1 = 105$ $\angle 1$ and $\angle 2$ form a linear pair.
$m\angle 3 = 105$ $\angle 3$ and $\angle 2$ form a linear pair.
$m\angle 4 = 75$ $\angle 4$ and $\angle 2$ are vertical angles.
$m\angle 5 = 105$ $\angle 5$ and $\angle 3$ are alternate interior angles.
$m\angle 6 = 75$ $\angle 6$ and $\angle 2$ are corresponding angles.
$m\angle 7 = 105$ $\angle 7$ and $\angle 3$ are corresponding angles.
$m\angle 8 = 75$ $\angle 8$ and $\angle 6$ are vertical angles.

Exercises

In the figure, $m\angle 3 = 102$. Find the measure of each angle.

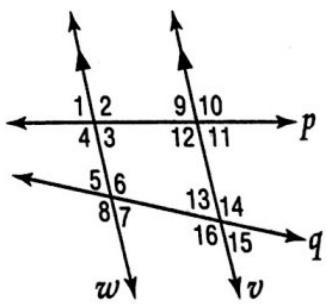


1. $\angle 5$ **2.** $\angle 6$

3. $\angle 11$ **4.** $\angle 7$

5. $\angle 15$ **6.** $\angle 14$

In the figure, $m\angle 9 = 80$ and $m\angle 5 = 68$. Find the measure of each angle.



7. $\angle 12$ **8.** $\angle 1$

9. $\angle 4$ **10.** $\angle 3$

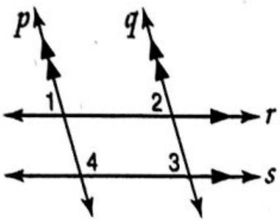
11. $\angle 7$ **12.** $\angle 16$

3-2 Study Guide and Intervention *(continued)*

Angles and Parallel Lines

Algebra and Angle Measures Algebra can be used to find unknown values in angles formed by a transversal and parallel lines.

Example **If $m\angle 1 = 3x + 15$, $m\angle 2 = 4x - 5$, $m\angle 3 = 5y$, and $m\angle 4 = 6z + 3$, find x and y.**



$p \parallel q$, so $m\angle 1 = m\angle 2$ because they are corresponding angles.

$$3x + 15 = 4x - 5$$
$$3x + 15 - 3x = 4x - 5 - 3x$$
$$15 = x - 5$$
$$15 + 5 = x - 5 + 5$$
$$20 = x$$

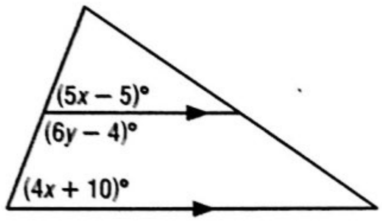
$r \parallel s$, so $m\angle 2 = m\angle 3$ because they are corresponding angles.

$$m\angle 2 = m\angle 3$$
$$75 = 5y$$
$$\frac{75}{5} = \frac{5y}{5}$$
$$15 = y$$

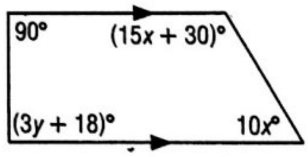
Exercises

Find x and y in each figure.

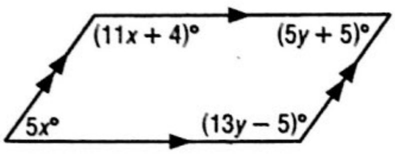
1.



2.



3.



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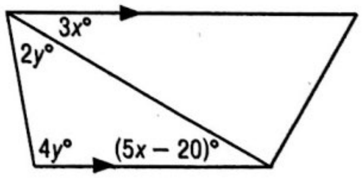
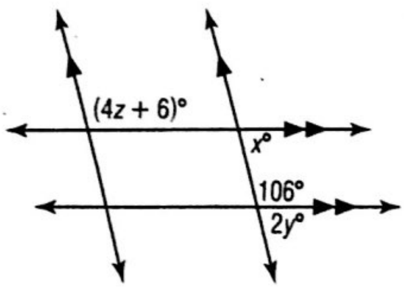
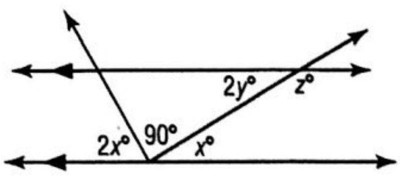


Find x, y, and z in each figure.

5.



6.

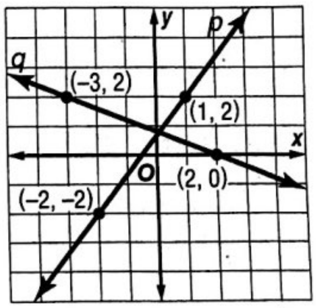


3-3 Study Guide and Intervention

Slopes of Lines

Slope of a Line The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

Example **Find the slope of each line.**



For line p, let (x_1, y_1) be $(1, 2)$ and (x_2, y_2) be $(-2, -2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2 - 2}{-2 - 1} \text{ or } \frac{4}{3}$$

For line q, let (x_1, y_1) be $(2, 0)$ and (x_2, y_2) be $(-3, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - 0}{-3 - 2} \text{ or } -\frac{2}{5}$$

Exercises

Determine the slope of the line that contains the given points.

1. $J(0, 0), K(-2, 8)$

2. $R(-2, -3), S(3, -5)$

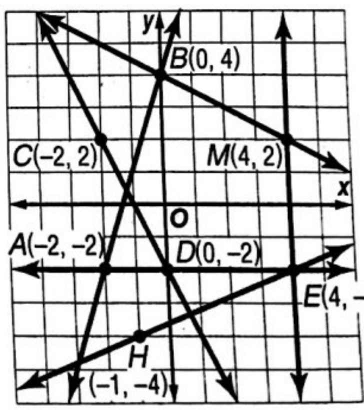
3. $L(1, -2), N(-6, 3)$

4. $P(-1, 2), Q(-9, 6)$

5. $T(1, -2), U(6, -2)$

6. $V(-2, 10), W(-4, -3)$

Find the slope of each line.



7. $\overleftrightarrow{AB}$

8. $\overleftrightarrow{CD}$

9. $\overleftrightarrow{EM}$

10. $\overleftrightarrow{AE}$

11. $\overleftrightarrow{EH}$

12. $\overleftrightarrow{BM}$

Slopes of Lines

Parallel and Perpendicular Lines If you examine the slopes of pairs of parallel lines and the slopes of pairs of perpendicular lines, where neither line in each pair is vertical, you will discover the following properties.

Two lines have the same slope if and only if they are parallel.

Two lines are perpendicular if and only if the product of their slopes is -1.

Example 1 **Find the slope of a line parallel to the line containing $A(-3, 4)$ and $B(2, 5)$.**

Find the slope of $\overleftrightarrow{AB}$. Use $(-3, 4)$ for (x_1, y_1) and use $(2, 5)$ for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 4}{2 - (-3)} \text{ or } \frac{1}{5}$$

The slope of any line parallel to $\overleftrightarrow{AB}$ must be $\frac{1}{5}$.

Example 2 **Find the slope of a line perpendicular to $\overleftrightarrow{PQ}$ for $P(-2, -4)$ and $Q(4, 3)$.**

Find the slope of $\overleftrightarrow{PQ}$. Use $(-2, -4)$ for (x_1, y_1) and use $(4, 3)$ for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3 - (-4)}{4 - (-2)} \text{ or } \frac{7}{6}$$

Since $\frac{7}{6} \cdot \left(-\frac{6}{7}\right) = -1$, the slope of any line perpendicular to $\overleftrightarrow{PQ}$ must be $-\frac{6}{7}$.

Exercises

Determine whether $\overleftrightarrow{MN}$ and $\overleftrightarrow{RS}$ are *parallel*, *perpendicular*, or *neither*.

1. $M(0, 3)$, $N(2, 4)$, $R(2, 1)$, $S(8, 4)$

2. $M(-1, 3)$, $N(0, 5)$, $R(2, 1)$, $S(6, -1)$

3. $M(-1, 3)$, $N(4, 4)$, $R(3, 1)$, $S(-2, 2)$

4. $M(0, -3)$, $N(-2, -7)$, $R(2, 1)$, $S(0, -3)$

5. $M(-2, 2)$, $N(1, -3)$, $R(-2, 1)$, $S(3, 4)$

6. $M(0, 0)$, $N(2, 4)$, $R(2, 1)$, $S(8, 4)$

Find the slope of $\overleftrightarrow{MN}$ and the slope of any line perpendicular to $\overleftrightarrow{MN}$.

7. $M(2, -4)$, $N(-2, -1)$

8. $M(1, 3)$, $N(-1, 5)$

9. $M(4, -2)$, $N(5, 3)$

10. $M(2, -3)$, $N(-4, 1)$

3-4

Study Guide and Intervention

Equations of Lines

Write Equations of Lines You can write an equation of a line if you are given any of the following:

- **the slope** and the y-intercept,
- **the slope** and the coordinates of a point on the line, or
- **the coordinates** of two points on the line.

If m is the slope of a line, b is its y-intercept, and (x_1, y_1) is a point on the line, then:

- **the slope-intercept form** of the equation is $y = mx + b$,
- **the point-slope form** of the equation is $y - y_1 = m(x - x_1)$.

Example 1 **Write an equation in slope-intercept form of the line with slope -2 and y-intercept 4.**

$y = mx + b$ Slope-intercept form

$y = -2x + 4$ $m = -2$, $b = 4$

The slope-intercept form of the equation of the line is $y = -2x + 4$.

Example 2 **Write an equation in point-slope form of the line with slope $-\frac{3}{4}$ that contains (8, 1).**

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 1 = -\frac{3}{4}(x - 8)$ $m = -\frac{3}{4}$, $(x_1, y_1) = (8, 1)$

The point-slope form of the equation of the line is $y - 1 = -\frac{3}{4}(x - 8)$.

Exercises

Write an equation in slope-intercept form of the line having the given slope and y-intercept.

1. m: 2, y-intercept: -3

2. m: $-\frac{1}{2}$, y-intercept: 4

3. m: $\frac{1}{4}$, y-intercept: 5

4. m: 0, y-intercept: -2

5. m: $-\frac{5}{3}$, y-intercept: $\frac{1}{3}$

6. m: -3, y-intercept: -8

Write an equation in point-slope form of the line having the given slope that contains the given point.

7. $m = \frac{1}{2}$, $(3, -1)$

8. $m = -2$, $(4, -2)$

9. $m = -1$, $(-1, 3)$

10. $m = \frac{1}{4}$, $(-3, -2)$

11. $m = -\frac{5}{2}$, $(0, -3)$

12. $m = 0$, $(-2, 5)$

NAME ______________________ DATE __________ PERIOD _____

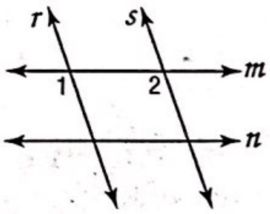
3-5 Study Guide and Intervention

Proving Lines Parallel

Identify Parallel Lines If two lines in a plane are cut by a transversal and certain conditions are met, then the lines must be parallel.

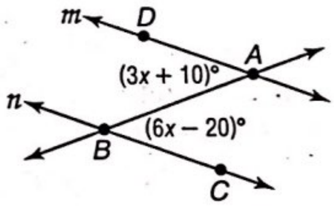
If	then
• corresponding angles are congruent, • alternate exterior angles are congruent, • consecutive interior angles are supplementary, • alternate interior angles are congruent, or • two lines are perpendicular to the same line,	the lines are parallel.

Example 1 **If $m\angle 1 = m\angle 2$, determine which lines, if any, are parallel.**



Since $m\angle 1 = m\angle 2$, then $\angle 1 \cong \angle 2$. $\angle 1$ and $\angle 2$ are congruent corresponding angles, so $r \parallel s$.

Example 2 **Find x and $m\angle ABC$ so that $m \parallel n$.**



We can conclude that $m \parallel n$ if alternate interior angles are congruent.

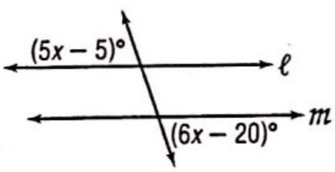
$$
\begin{aligned}
m\angle BAD &= m\angle ABC \\
3x + 10 &= 6x - 20 \\
10 &= 3x - 20 \\
30 &= 3x \\
10 &= x
\end{aligned}
$$

$$
\begin{aligned}
m\angle ABC &= 6x - 20 \\
&= 6(10) - 20 \text{ or } 40
\end{aligned}
$$

Exercises

Find x so that $\ell \parallel m$.

1.



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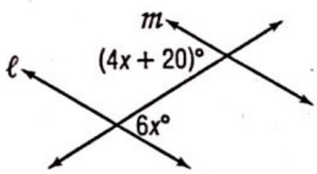

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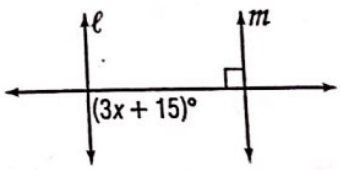

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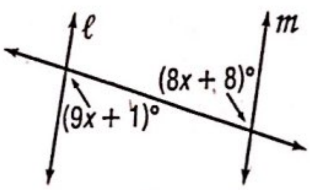

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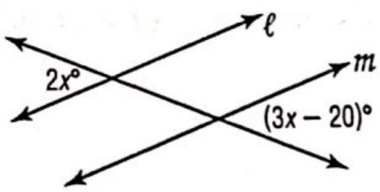

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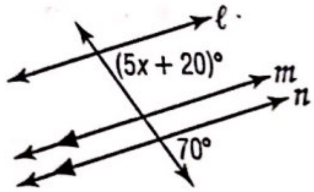


NAME ________________ DATE ________ PERIOD ____

3-5 Study Guide and Intervention *(continued)*

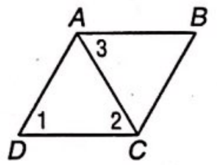
Proving Lines Parallel

Prove Lines Parallel You can prove that lines are parallel by using postulates and theorems about pairs of angles. You also can use slopes of lines to prove that two lines are parallel or perpendicular.

Example

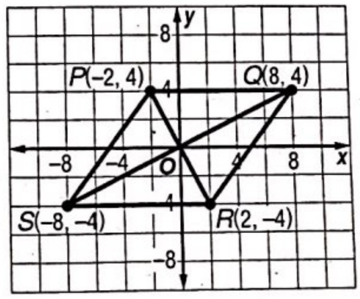
a. Given: $\angle 1 \cong \angle 2$, $\angle 1 \cong \angle 3$

Prove: $\overline{AB} \parallel \overline{DC}$



Statements	Reasons
1. $\angle 1 \cong \angle 2$ $\angle 1 \cong \angle 3$	1. Given
2. $\angle 2 \cong \angle 3$	2. Transitive Property of $\cong$
3. $\overline{AB} \parallel \overline{DC}$	3. If alt. int. angles are $\cong$, then the lines are $\parallel$.

b. Which segments are parallel? Which segments are perpendicular?



slope of $\overline{PQ} = 0$ slope of $\overline{SR} = 0$

slope of $\overline{PS} = \frac{4}{3}$ slope of $\overline{QR} = \frac{4}{3}$

slope of $\overline{PR} = -2$ slope of $\overline{SQ} = \frac{1}{2}$

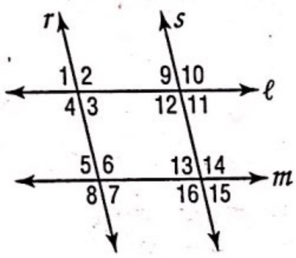
So $\overline{PQ} \parallel \overline{SR}$, $\overline{PS} \parallel \overline{QR}$, and $\overline{PR} \perp \overline{SQ}$.

Exercises

For Exercises 1–6, complete the proof.

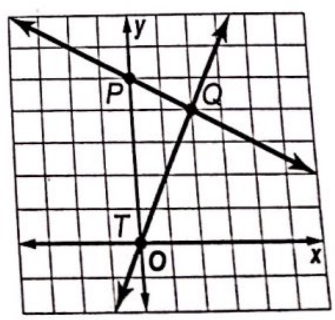
Given: $\angle 1 \cong \angle 5$, $\angle 15 \cong \angle 5$

Prove: $\ell \parallel m$, $r \parallel s$



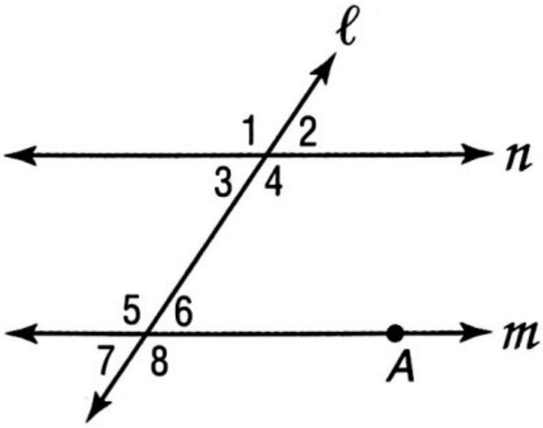
Statements	Reasons
1. $\angle 15 \cong \angle 5$	1. ________________
2. $\angle 13 \cong \angle 15$	2. ________________
3. $\angle 5 \cong \angle 13$	3. ________________
4. $r \parallel s$	4. ________________
5. ________________	5. Given
6. ________________	6. If corr $\angle$s are $\cong$, then lines $\parallel$.

7. Determine whether $\overleftrightarrow{PQ} \perp \overleftrightarrow{TQ}$. Explain why or why not.



Vocabulary Check

Refer to the figure and choose the term that best completes each sentence.



1. Angles 4 and 5 are (consecutive, alternate) interior angles.
2. The distance from point A to line n is the length of the segment (perpendicular, parallel) to line n through A.
3. If $\angle 4$ and $\angle 6$ are supplementary, lines m and n are said to be (parallel, intersecting) lines.
4. Line ℓ is a (slope-intercept, transversal) for lines n and m.
5. $\angle 1$ and $\angle 8$ are (alternate interior, alternate exterior) angles.
6. If $n \parallel m$, $\angle 6$ and $\angle 3$ are (supplementary, congruent).
7. Angles 5 and 3 are (consecutive, alternate) interior angles.
8. If $\angle 2 \cong \angle 7$, then lines n and m are (skew, parallel) lines.

Lesson-by-Lesson Review

3-1 Parallel Lines and Transversals (pp. 142–147)

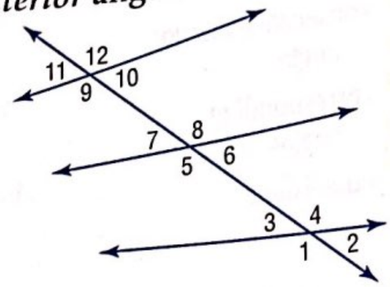
Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.

9. $\angle 3$ and $\angle 6$

10. $\angle 5$ and $\angle 3$

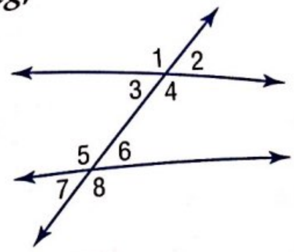
11. $\angle 2$ and $\angle 7$

12. $\angle 4$ and $\angle 8$



13. EAGLES The flight paths of two American bald eagles were tracked at an altitude of 8500 feet in a direction north to south and an altitude of 12,000 feet in a direction west to east, respectively. Describe the types of lines formed by the paths of these two eagles. Explain your reasoning.

Example 1 **Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior* angles.**



a. $\angle 7$ **and** $\angle 3$ corresponding

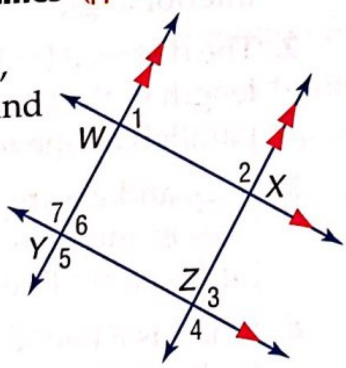
b. $\angle 4$ **and** $\angle 6$ consecutive interior

c. $\angle 7$ **and** $\angle 2$ alternate exterior

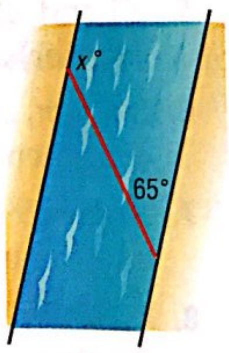
d. $\angle 3$ **and** $\angle 6$ alternate interior

3-2 Angles and Parallel Lines (pp. 149–154)

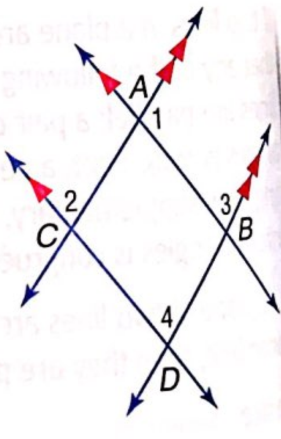
14. If $m\angle 1 = 3a + 40$, $m\angle 2 = 2a + 25$, and $m\angle 3 = 5b - 26$, find a and b.



15. BOATING To cross the river safely, Georgia angles her canoe 65° from the river's edge, as shown. At what angle x will she arrive on the other side of the river?



Example 2
If $m\angle 1 = 4p + 15$, and $m\angle 3 = 3p - 10$, find p.



Since $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$, $\angle 1$ and $\angle 3$ are supplementary by the Consecutive Interior Angles Theorem.

$$m\angle 1 + m\angle 3 = 180 \quad \text{Def. of suppl. } \angle s$$
$$(4p + 15) + (3p - 10) = 180 \quad \text{Substitution}$$
$$7p + 5 = 180 \quad \text{Simplify.}$$
$$7p = 175 \quad \text{Subtract.}$$
$$p = 25 \quad \text{Divide.}$$

3-3 Slopes of Lines (pp. 156–163)

Graph the line that satisfies each condition.

16. contains (2, 3) and is parallel to $\overleftrightarrow{AB}$ with $A(-1, 2)$ and $B(1, 6)$

17. contains $(-2, -2)$ and is perpendicular to $\overleftrightarrow{PQ}$ with $P(5, 2)$ and $Q(3, -4)$

18. PAINTBALL During a game of paintball, Trevor and Carlos took different paths. If the field can be mapped on the coordinate plane, Trevor ran from $(-5, -3)$ to $(4, 3)$ and Carlos from $(2, -7)$ to $(-6, 5)$. Determine whether their paths are *parallel*, *perpendicular*, or *neither*.

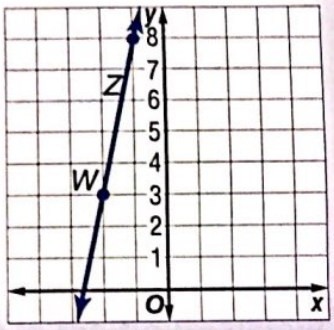
Example 3 **Graph the line that contains $W(-2, 3)$ and is parallel to $\overleftrightarrow{XY}$ with $X(3, -4)$ and $Y(5, 6)$.**

slope of $\overleftrightarrow{XY} = \frac{6-(-4)}{5-3} = \frac{10}{2} = 5$

The slope of the line parallel to $\overleftrightarrow{XY}$ through $W(-2, 3)$ is also 5, since parallel lines have the same slope.

Graph the line. Start at $(-2, 3)$. Move up 5 units and then move right 1 unit. Label the point Z.

Draw $\overleftrightarrow{WZ}$.



3-4 Equations of Lines (pp. 165–170)

Write an equation in the indicated form of the line that satisfies the given conditions.

19. $m = 2$, contains $(1, -5)$; point-slope

20. $m = -\frac{3}{2}$, contains $(2, -4)$; slope-intercept

21. contains $(-3, -7)$ and $(9, 1)$; point-slope

22. contains $(2, 5)$ and $(-2, -1)$; slope-intercept

23. DRIVING A car traveling at 30 meters per second begins to slow down or *decelerate* at a constant rate. After 2 seconds, its velocity is 16 meters per second. Write an equation that represents the car's velocity v after t seconds. Then use this equation to determine how long it will take the car to come to a complete stop.

Example 4 **Write an equation in slope-intercept form of the line that passes through $(2, -4)$ and $(-3, 1)$.**

Find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope Formula}$$

$$= \frac{1-(-4)}{-3-2} \qquad (x_1, y_1) = (2, -4),\ (x_2, y_2) = (-3, 1)$$

$$= \frac{5}{-5} \text{ or } -1 \qquad \text{Simplify.}$$

Now use the point-slope form and either point to write an equation.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - (-4) = -1(x - 2) \qquad m = -1, (x_1, y_1) = (2, -4)$$

$$y + 4 = -x + 2 \qquad \text{Simplify.}$$

$$y = -x - 2 \qquad \text{Subtract 4 from each side.}$$