Find the sum of the measures of the interior angles of each convex polygon.

1. hexagon

SOLUTION:

A hexagon has six sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute n = 6 in (n-2)180.

$$(n-2)180 = (6-2)180$$

= $4 \cdot 180$
= 720

2. 16-gon

SOLUTION:

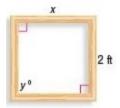
A 16-gon has sixteen sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute n = 16 in (n-2)180.

$$(n-2)180 = (16-2)180$$

= $14 \cdot 180$
= 2520

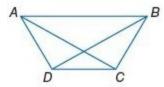
- 3. **ART** Jen is making a frame to stretch a canvas over for a painting. She nailed four pieces of wood together at what she believes will be the four vertices of a square.
 - **a.** How can she be sure that the canvas will be a square?
 - **b.** If the canvas has the dimensions shown below, what are the missing measures?



SOLUTION:

- **a.** She should measure the angles at the vertices to see if they are 90 or she can check to see if the diagonals are congruent and perpendicular.
- **b.** Each side of a square has the same measure, so x = 2 ft. Each angle of a square is a right angle, so y = 90.

Quadrilateral ABCD is an isosceles trapezoid.



4. Which angle is congruent to $\angle BCD$?

SOLUTION:

The base angles of an isosceles triangle are congruent, so $\angle ADC$ is congruent to $\angle BCD$.

5. Which side is parallel to \overline{AB} ?

SOLUTION:

The bases of a trapezoid are parallel, so \overline{DC} is parallel to \overline{AB} .

6. Which segment is congruent to \overline{AC} ?

SOLUTION:

The diagonals of an isosceles trapezoid are congruent, so \overline{BD} is congruent to \overline{AC} .

The measure of the interior angles of a regular polygon is given. Find the number of sides in the polygon.

7.900

SOLUTION:

Let n be the number of sides in the polygon. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as (n-2)180.

$$900 = (n-2)180$$

$$900 = 180n - 360$$

$$180n = 1260$$

$$n = 7$$

8. 1980

SOLUTION:

Let n be the number of sides in the polygon. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as (n-2)180.

$$1980 = (n-2)180$$

$$1980 = 180n - 360$$

$$180n = 2340$$

$$n = 13$$

9.2880

SOLUTION:

Let n be the number of sides in the polygon. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as (n-2)180.

$$2880 = (n-2)180$$

$$2880 = 180n - 360$$

$$180n = 3240$$

$$n = 18$$

10.5400

SOLUTION:

Let n be the number of sides in the polygon. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as (n-2)180.

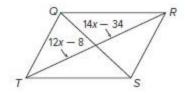
$$5400 = (n-2)180$$

$$5400 = 180n - 360$$

$$180n = 5760$$

$$n = 32$$

11. **MULTIPLE CHOICE** If *QRST* is a parallelogram, what is the value of x?



A 11

B 12

C 13

D 14

SOLUTION:

Diagonals of a parallelogram bisect each other. So, 14x - 34 = 12x - 8.

Solve for x.

$$14x - 34 = 12x - 8$$
 Diag. bisect each other.

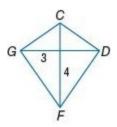
$$2x - 34 = -8$$
 Subtract $12x$ from each side.

$$2x = 26$$
 Add 34 to each side.
 $x = 13$ Divide each side by 2.

So, the correct answer is choice C.

If CDFG is a kite, find each measure.

12. *GF*



SOLUTION:

Use the Pythagorean Theorem.

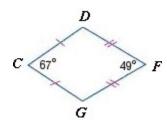
$$GF^2 = 3^2 + 4^2$$

$$GF = \pm \sqrt{9 + 16}$$

$$GF = \pm 5$$

Since the length must be positive, GF = 5.

13. *m*∠*D*



SOLUTION:

Since a kite can only have one pair of opposite congruent angles and $\angle C \ncong \angle F$, then $\angle D \cong \angle G$. Let x be the measure of $\angle D$.

The sum of the measures of the angles of a quadrilateral is 360.

$$m\angle C + m\angle D + m\angle F + m\angle G = 360$$

$$67 + x + 49 + x = 360$$

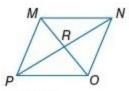
$$2x + 116 = 360$$

$$2x = 244$$

$$x = 122$$

So,
$$m \angle D = m \angle G = 122$$
.

ALGEBRA Quadrilateral MNOP is a rhombus. Find each value or measure.



14. *m∠MRN*

SOLUTION:

Since the diagonals of a rhombus are perpendicular, $m \angle MRN = 90$ by the definition of perpendicular lines.

15. If PR = 12, find RN.

SOLUTION:

In a rhombus, diagonals bisect each other. So, RN = PN = 12.

16. If $m\angle PON = 124$, find $m\angle POM$.

SOLUTION:

Since MNOP is a rhombus, diagonal \overline{OM} bisects $\angle PON$. Therefore, $m \angle POM = \frac{1}{2}m \angle PON$.

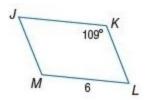
$$m\angle POM = \frac{1}{2}m\angle PON$$
$$= \frac{1}{2}(124)$$
$$= 62$$

17. **CONSTRUCTION** The Smiths are building an addition to their house. Mrs. Smith is cutting an opening for a new window. If she measures to see that the opposite sides are congruent and that the diagonal measures are congruent, can Mrs. Smith be sure that the window opening is rectangular? Explain.

SOLUTION:

Yes, that is enough to show that the opening is a rectangle. Since each pair of opposite sides are the same length, the opening is a parallelogram. By Theorem 6.14, if the diagonals of a parallelogram are congruent then it is a rectangle.

Use JKLM to find each measure.



18. *m∠JML*

SOLUTION:

Opposite angles of a parallelogram are congruent.

So, $m \angle JML = 109$.

19. *JK*

SOLUTION:

Opposite sides of a parallelogram are congruent.

So,
$$JK = 6$$
.

20. *m∠KLM*

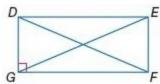
SOLUTION:

Consecutive angles in a parallelogram are supplementary.

So,
$$m \angle KLM + 109 = 180$$
.

$$m\angle KLM = 71$$

ALGEBRA Quadrilateral DEFG is a rectangle.



21. If DF = 2(x + 5) - 7 and EG = 3(x - 2), find EG.

SOLUTION:

The diagonals of a rectangle are congruent to each other. So, DF = EG.

$$2(x+5)-7=3(x-2)$$

 $2x+10-7=3x-6$

$$-x+3=-6$$
$$-x=-9$$

$$x = 9$$

Use the value of *x* to find *EG*.

$$EG = 3(9-2) = 21$$

22. If $m\angle EDF = 5x - 3$ and $m\angle DFG = 3x + 7$, find $m\angle EDF$.

SOLUTION:

$$m\angle EDF = m\angle DFG$$

$$5x - 3 = 3x + 7$$

$$2x = 10$$

$$x = 5$$

Substitute x = 5 in $m \angle EDF$.

$$m\angle EDF = 5(5) - 3$$

$$=25-3$$

$$= 22$$

23. If DE = 14 + 2x and GF = 4(x - 3) + 6, find GF.

SOLUTION:

Opposite sides of a rectangle are congruent. So, DE = GF.

$$14 + 2x = 4(x - 3) + 6$$
 Substitute.

$$14 + 2x = 4x - 12 + 6$$
 Distributive Property

$$14 + 2x = 4x - 6$$
 Simplify.

$$14 - 2x = -6$$
 Subtract $4x$ from each side.

$$-2x = -20$$
 Subtract 14 from each side.

$$2x = 20$$
 Divide each side by -1.

$$x = 10$$
 Divide each side by 2.

Use the value of x to find GF.

$$GF = 4(x - 3) + 6$$
 Original equation

$$=4(10-3)+6$$

Substitute.

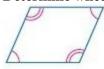
$$=4(7)+6$$

Subtract.

$$= 3\dot{4}$$

Simplify.

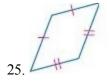
Determine whether each quadrilateral is a parallelogram. Justify your answer.



SOLUTION:

24.

Each pair of opposite angles are congruent. By Theorem 6.10, if both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. No other information is needed to determine that the figure is a parallelogram.



SOLUTION:

There are 2 pairs of consecutive sides that are congruent. Since opposite sides are not congruent, this fails Theorem 6.9. If both pairs of opposite sides are congruent, the quadrilateral is a parallelogram. This is not a parallelogram.